Electrostatic solitary waves containing electron beam featuring q-nonextensive distribution of electrons

Sheenu Juneja[1], Parveen Bala[2]
[1, 2] Department of Mathematics, Statistics & Physics
[1, 2] Punjab Agricultural University, Ludhiana-141004

Abstract- *In the present investigation, small amplitude electrostatic solitary waves have been studied in an environment of cold positive, negative ions with an electron beam along with electrons obeying q-nonextensive distribution. Using reductive perturbation method, a fourth degree dispersion relation has been derived that corresponds to four modes out of which three are slow and fourth one is fast. Depending upon the values of parameters used either compressive or rarefactive solitons are obtained corresponding to these modes. The effects of nonextensive parameter and beam parameters have been discussed for the soliton profile of all the modes. The numerical results have been presented in the form of two and three dimensional plots for the three ranges of nonextensive parameters q i.e. $-1<q<0$, $0<q<1$ and $q>1$ respectively.*

Keywords- Nonextensive distribution, Reductive Perturbation method, Electrostatic solitary waves, Korteweg-de Vries (KdV) equation, Electron beam

I. INTRODUCTION

Nonlinearity plays an important role in plasma dynamics, because equations in plasma dynamics are nonlinear. The balance of nonlinearity, dispersion and dissipation results in some nonlinear structures like solitons, vortices and double layers etc [1]. Out of these, solitons have been a source of attraction for researchers as they offer a rich physical insight underlying the nonlinear phenomenon. Solitons exhibit particle like properties because energy at any instant confined to a limited region of space were proposed as models for elementary particles [2]. A wave packet in which the wavefield is localized in a limited generally propagating spatial region and absent outside the region is called a soliton. These solitary waves are special case of the solution of partial differential equation which are called travelling waves [3]. Some form of perturbation methods are adopted to study nonlinear wave structures that end up in deriving some nonlinear partial differential equations like Korteweg-de-Vries (KdV), modified Korteweg-de-Vries (m-KdV) and Schrodinger equations which belong to a large class of nonlinear evolution equations [1]. Zabusky and Kruskal [3] found that stable pulse-like waves could exist in a system described by KdV equation. A remarkable quantity of these solitary waves was that they collide with each other and yet preserve their shapes and speeds after the collision.The solitary waves were first experimentally observed by Ikezi et al [4]. The reductive perturbation method (RPM) was first used by Washimi and Taniuti [5] to drive KdV equation. Ion acoustic solitary waves (IASWs) have been invesigated for several decades both theoretically [6-16] and experimentally [17-21].

Space plasma are observed to be of multicomponent type including positive and negative ions. Uberoi and Das [22] predicted that plasma may consist of positive and negative ions in lower atmosphere. In D-region of ionosphere [23-24] the multi-ion plasmas i.e. $H^+O_2^-$, H^+H^- and Ar^+F^- etc are found. However, negative ions in a plasma have dominant role on the formation of ion-acoustic solitary waves [9]. Many researchers have used multicomponent plasma containing negative ions in their study [10,11,17,19,20,25]. Electron beam component is frequently observed in the region of space where ion-acoustic waves exist [26]. The presence of electron beam in a plasma system results in modification of the properties and conditions for the existence of electrostatic exications [27-29]. Further the invesigation of electron beam system has also considerable importance in the areas of magnetosphere and solar physics [30]. Electrostatic waves in a beam plasma system were studied by a number of researchers [31-35]. Yadav et al [36] showed that the four soliton branches exist above the critical beam velocity in an electron beam plasma system. However, Bala et al [37] showed that in mulicomponent plasma model which contained positive, negative ions, electron beam and nonthermal electrons, six soliton branches are formed above the critical beam velocity. Labany et al [38] studied the nonlinear properties of electrostatic waves in a strongly magnetized beam plasma system consisting of hot and cold electron fluid with vortex type electron distribution. They extended their study to include higher order contributions as well. Recently Shan and Saleem [29] studied small amplitude ion-acoustic double layers with electron beam and q-nonextensive electrons. However they considered only positive ions in their study. In the present investigation, we have included negative ions as well. Devanandhan et al [39] theoretically analysed the electron-acoustic solitons in beam plasma assuming electrons to be superthermal where they found the ranges of amplitude and width of nonlinear waves. It may be mentioned that owing to their tedious nature, investigations of small amplitude ion-acoustic waves in beam plasma system are few in number.

A significant increase in richness and variety of wave motions are provided by particle distributions, these are categorized as Maxwellian and non-Maxwellian. Maxwellian distribution also known as extensive distribution, is valid universally for systems that are in equilibrium. But this distribution is not applicable to describe correctly the systems with long range interaction, long time memory, fractality of the corresponding space-time/phase-space, etc. For long range interaction such as in plasma, non-equilibrium stationary states i.e. nonextensive distribution exists. Since last two decade there is an increasing focus on a new statistical approach known as Tsallis distribution [40], this is the topic of interest due to its relevance in astrophysical and cosmological scenario. Non-extensive effects on ion acoustic waves are not apparent when electron temperature is much more than the ion temperature, but they are salient when the electron temperature is much more than ion temperature. As compared with the electrons, the ions play a dominant role in the non-extensive effects [41]. Lima et al [42] have found that the q-nonextensive pattern is very important for systems having long range interactions (i.e studying interactions comparable with the size of the system under consideration) such as those occures in astrophysics and plasma physics. Here, the parameter q is a measure of nonextensitivity and q=1 leads to extensive distribution. For q<1(>1), high energy states are more (less probable than in extensive case. The Tasllis q-distribution has been used with some success in a number of research work in plasma physics [43-55]. The aim of present investigation is to study ion-acoustic solitary waves in multi-ion plasma model consisting of positive, negative ion and electron beam with nonextensitivily distributed electrons. It is important to mention that the present study is an extension of author's earlier work [56], where they presented a detailed analysis of critical beam velocity in the environment of positive, negative ions and electron beam. Regarding the organization of paper, in this section II. basic equations related to our plasma model have been given and standard reductive perturbation method, KdV equation has been derived and also soliton solution is given in section II. Section III. is devoted to the discussion of numerical results and finally the conclusions are made.

II. BASIC EQUATIONS AND DERIVATION OF KDV EQUATION

Let us considered a collisionless multicomponent plasma model composed of cold positive and negative ions and electron beam with nonextensively distributed electrons in the absence of external magnetic field. The nonlinear behavior of such system may be described by the following set of normalized continuity, momentum and Poission equations [56]:

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j v_j)}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} = \frac{1}{Q_j}\frac{\partial \phi}{\partial x} \qquad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \alpha_b n_b - \frac{n_1}{\rho} + \frac{\alpha\varepsilon_z}{\rho} n_2 \qquad (3)$$

where, j=1,*2*,*b*, 1 stands for positive ions, 2 stands for negative ions, b stands for electron beam. Here, $Q_1 = -\delta$, $Q_2 = \eta\delta/\varepsilon_z$, $Q_b = \eta$

$$\rho = (1-\alpha\varepsilon_z)/(1+\alpha_b), \alpha_b = \frac{n_b^{(0)}}{n_e^{(0)}}, \alpha = \frac{n_2^{(0)}}{n_1^{(0)}}, \eta = \frac{m_2}{m_1},$$

$$\eta_e = \frac{m_e}{m_1}, \varepsilon_z = \frac{Z_2}{Z_1}, \delta = \frac{1+\alpha\varepsilon_z^2/\eta + \alpha_b/\eta_e Z_1}{\rho}$$

Here $(n_1, v_1), (n_2, v_2)$ and (n_b, v_b) are the densities and fluid velocities of positive and negative ion species and electron beam respectively. ${n_1}^{(0)}, {n_2}^{(0)}, {n_b}^{(0)}$ are the equilibrium densities of two ion components and beam respectively. In equations (1) to (3), velocities (v_1, v_2, v_b), ϕ (potential), time (t) and space coordinates (x) have been normalized with respect to the ion- acoustic speed in the mixture, $C_s = (T_e \delta Z_1/m_1)^{\frac{1}{2}}$, thermal potential T_e/e, inverse of ion plasma frequency is the mixture ${\omega_{pi}}^{-1} = (m_1/4\pi n_{e0}\delta Z_1)^{\frac{1}{2}}$ and Debye length $\lambda_D = (T_e/4\pi n_{eo}e^2)^{\frac{1}{2}}$ respectively. Ion densities (n_1, n_2, n_b) are normalized with their corresponding equilibrium values, whereas electron densities are normalized by ${n_e}^{(0)}$. The number density of electron fluid, with nonextensive distribution is given by [40]:

$$n_e = [1 + (q-1)\phi]^{\frac{(q+1)}{2(q-1)}} \qquad (4)$$

Here, q is the nonextensive patameter, known as entropy index.

To study small but finite amplitude ion acoustic solitary waves in our multispecies plasma model, we construct here a weakly nonlinear theory of ion-acoustic waves which leads to scaling of the independent variables through the stretched co-ordinates $\xi = \varepsilon^{\frac{1}{2}}(x - \lambda t)$ and $\tau = \varepsilon^{3/2} t$. Where ε is small parameter measuring the weakness of the dispersion and λ is the phase velocity of wave. Now to strike balance between nonlinear and dispersive terms, we use reductive perturbation technique where we expand all dependent quantities in equations (1) to (3) around the equilibrium values in power of ε in the following form:

$$\begin{pmatrix} n_j \\ v_j \\ \phi \end{pmatrix} = \begin{pmatrix} 1 \\ k \\ 0 \end{pmatrix} + \sum_{r=1}^{\infty} \varepsilon^r \begin{pmatrix} n_j^{(r)} \\ v_j^{(r)} \\ \phi^{(r)} \end{pmatrix} \tag{5}$$

Here *k=0* for positive and negative ions and $k = v_0$ for electron beam, is the initial electron beam velocity. Using stretched co-ordinates and equation (5) into Poisson's equation (3), to the lowest order of ε, we get the following dispersion relation, which is a polynomial of fourth degree in λ [56].

$$\frac{(\eta + \alpha\varepsilon_z^2)}{\eta\delta\lambda^2 Z_1\rho} + \frac{\alpha_b}{(v_0-\lambda)^2\eta_e\delta Z_1} = \frac{(q+1)}{2} \tag{6}$$

To the next power in ε, after a long algebraic but straight forward manipulations, the following nonlinear equation known as Korteweg-de Vries (KdV) equation is obtained.

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0 \tag{7}$$

Where *A=2Q/P* is nonlinearity coefficient and *B=2/P* is dispersion coefficient.

Where, $$P = \frac{4\delta\lambda Z_1^2}{\rho}\left(\frac{\eta + \alpha\varepsilon_z^2}{(\eta\delta Z_1\lambda^2)^2}\right) - \frac{4\alpha_b(v_0-\lambda)}{(\eta_e\delta Z_1(v_0-\lambda)^2)^2} \tag{8}$$

$$Q = \left(\frac{3\eta\delta\lambda^2(1-\alpha\varepsilon_z^3)}{\rho(\eta\delta\lambda^2)^3}\right) - \frac{\alpha_b 3\eta_e\delta(v_0-\lambda)^2}{(\eta_e\delta(v_0-\lambda)^2)^3} + \frac{(3-q)(q+1)}{4} \tag{9}$$

The steady state solution of KdV equation (7) is obtained by transforming the independent variables ξ and τ as

$$\chi = \xi - u\tau \tag{10}$$

Where u is normalized constant velocity.

Using boundary conditions $\chi \to \pm\infty, (\phi, \frac{\partial\phi}{\partial\chi}, \frac{\partial^2\phi}{\partial\chi^2}) \to 0$,

we get the condition uP>0. Accordingly, soliton velocity u will take either positive or negative values. Here we discuss three different ranges of the entropic index q for four soliton branches $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively. For mode λ_1 and λ_3, u is negative while λ_2 and λ_4, u is positive.

The soliton solution for equation (7) is given by

$$\phi = \phi_0 Sech^2(\frac{\chi}{w}) \tag{11}$$

Where the amplitude ϕ_0 and width *w* are given by

$$\phi_0 = \frac{3Pu}{Q}, \quad w = \sqrt{\frac{4}{uP}}$$

Peak amplitude and width are functions of parameters like nonextensivity (*q*), beam velocity and beam density.

III. DISCUSSION OF NUMERICAL RESULTS

Present investigation is dedicated to the numerical study of effects of beam parameters such as beam density (α_b), beam velocity v_o and entropy index (*q*) on the propagation characteristics of ion-acoustic solitary waves in a multicomponent beam plasma system. The electrons are considered to obey q-nonextensive distribution. As stated earlier, the dispersion relation is a fourth degree polynomial in λ. Correspondingly four modes propagating with different phase velocities exist where beam velocity is taken as greater than certain value of critical velocity. It must be mentioned here that, the numerical study of the critical beam velocity was presented in previous study by the same authors [56] and the present work is the extension of that earlier work. We will take the value of $v_o \approx 2.5$ for our study for which four solitons branches exist. The subscripts 1,2,3,4 refer to modes $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively. From numerical computation, three slow ion-acoustic modes have been observed with phase velocity less than beam velocity whereas the fourth one (λ_4) has phase velocity larger than beam velocity. This mode corresponds to fast mode.

A. For the Range -1<q<0

For solitons propagating in negative direction i.e. λ_1, peak amplitude as a function of beam density α_b has been given in figure 1(a), for three different values of *q*, for Ar^+F^- (η=0.476) plasma system with v_o = 2.5 and α = 0.1. A transition from rarefactive to compressive solitons result with

α_b. However this transition occur at higher values of beam density with decrease in q.

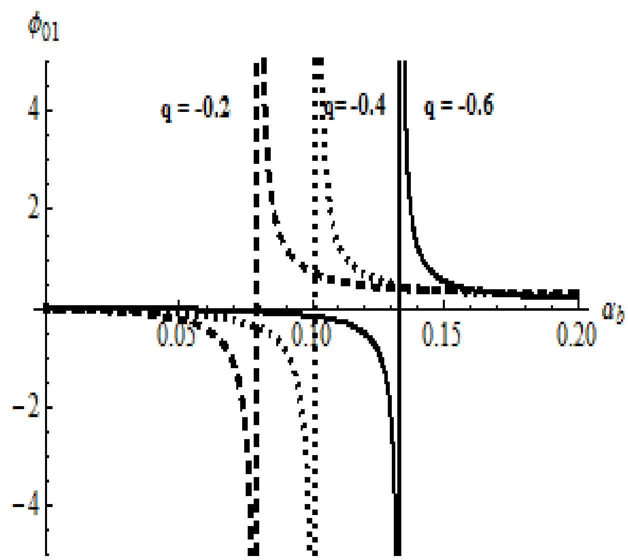

Figure 1(a). For the mode λ_1, -1<q<0, plot of amplitude ϕ_{01} as a function of α_b for three different value of q, with η= 0.476, u = -0.01, v_o= 2.5, α=0.1

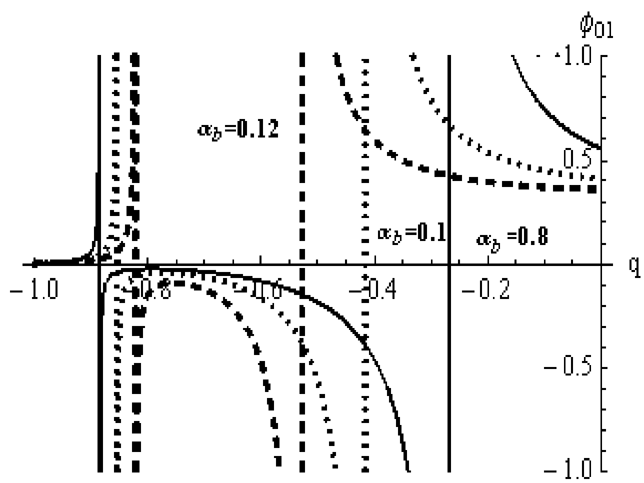

Figure 1(b). For the mode λ_1, (-1<q<0), plot of amplitude ϕ_{01} as a function of q for three different value of α_b, with other parameters of figure 1(a).

Further to investigate the effect of entropy index *q* on the soliton behavior, a plot of peak amplitude of mode λ_1, as a function of *q* has been given in figure1(b) for three different values of α_b. For the given beam density, there are two critical values of *q* i.e. q_{c1} and q_{c2} at which transition occurs from compressive□rarefactive□compressive (i.e. *C□R□C*) solitons with *q*. For $q<q_{c1}$ ($q>q_{c2}$), peak amplitude of compressive soliton increases (decreases) with q, while for $q_{c1}<q<q_{c2}$, peak amplitude of rarefactive solitons first increases and then decreases for higher *q*. It may be mentioned that for other plasma systems viz. H^+H^- and $H^+O_2^-$, similar behavior has been observed (not shown here).

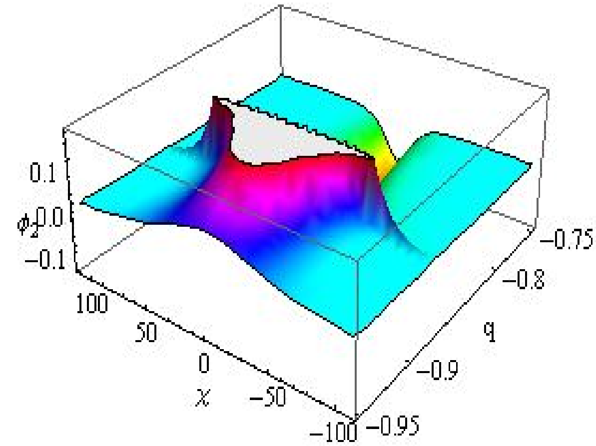

Figure 1(c). For the mode λ_2, -1<q<0, plot of soliton solution ϕ_2 as a function of χ and nonextensive parameter q with α=0.1, v_o= 2.5, u=0.01, η=0.476, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

However for mode λ_2, there is only one critical value of q at which transition occurs. The behavior is represented by 3D graph in figure 1(c) where soliton solution ϕ_2 has been plotted as a function of q and χ for $Ar^+ F^-$ plasma system. For mode λ_3, there is certain value of q below which no soliton exists and this value depends upon the beam density α_b.

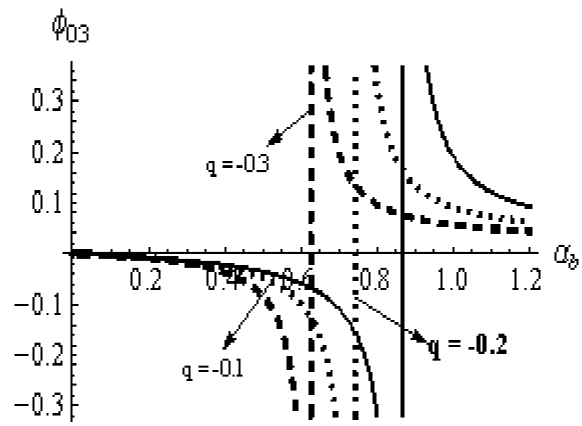

Figure 1(d). For the mode λ_3, -1<q<0, plot of amplitude ϕ_{03} as a function of beam density α_b, for different three values of q with η= 0.476, u = -0.01, v_o= 2.5, α= 0.1, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

For q>-0.5, plot of ϕ_{03} as a function of α_b has been given in figure 1(d), for three different values of nonextensive parameter q. Like modes λ_1 and λ_2, there is transition from rarefactive to compressive solitons. However, contrary to modes λ_1 and λ_2, this transition occurs at smaller values of beam density (α_b). Similar behavior has been reported for mode λ_4 (not shown here).

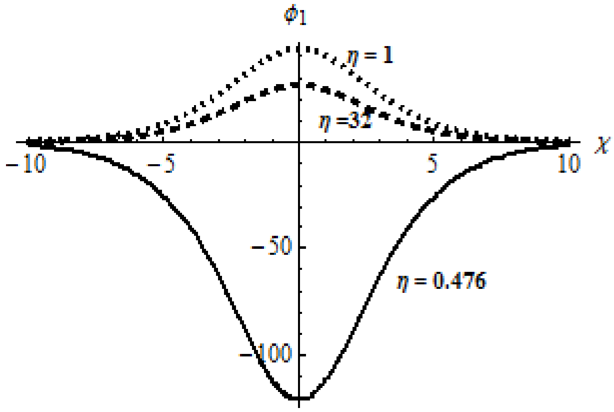

Figure 1(e). For the mode λ_1, (-1<q<0), plot of soliton solution ϕ_1 as a function of χ, for three different value of η with u = -0.01, v_o= 2.5, α= 0.1, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

For given beam density α_b, beam velocity v_o and nonextensive parameter q, mass of negative ions modify the behavior of solitons. For mode λ_1 plot of soliton solution ϕ_1, as a function of χ has been given in figure1(e).

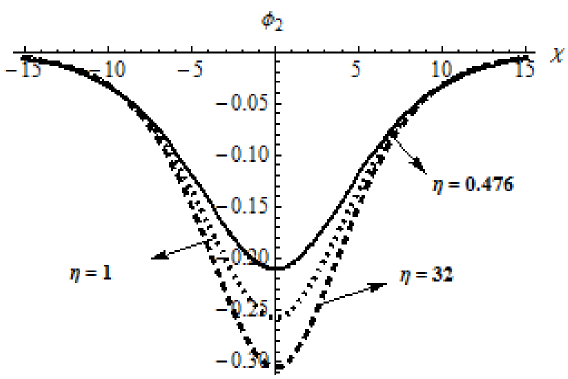

Figure 1(f). For the mode λ_2, (-1<q<0), plot of soliton solution ϕ_2 as a function of χ, for three different value of η with α=0.1, v_o=2.5, u=0.01, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

For given α_b, a transition from rarefactive to compressive soliton result with increase in mass of negative ions and amplitude of compressive solitons decreases with η. However from similar plot for mode λ_2, only rarefactive solitons result, where amplitude decreases with η (see figure 1(f)). A similar behavior has been observed for mode λ_3 (not shown), while for mode λ_4, it is observed that mass of negative ion has no effect on the amplitude of rarefactive soliton.

B. For the Range 0<q<1

For this range of nonextensitivity, for solitons branch λ_1, a plot of peak amplitude ϕ_{01} as a function of *q* has been given in figure 2(a), for three different value of beam density α_b. Again a Transition from rarefactive to compressive soliton occurs. For larger value of beam density α_b, transition occurs at lower values of *q*. Peak amplitude of rarefactive as well as compressive soliton is found to decrease with q. Similar behavior has been reported for mode λ_2. However only rarefactive soliton are observed for mode modes λ_3 and λ_4, for which peak amplitude decreases with α_b (not shown here). For this range of *q*, from the plot of soliton solution ϕ_1, as a function of χ, (figure 2(b)), peak amplitude was observed to decrease with increase in mass of negative ions. An analogous behavior has been reported for modes λ_2. However for modes λ_3 and λ_4, compressive solitons are obtained and mass of negative ion has no effect on amplitude (not shown).

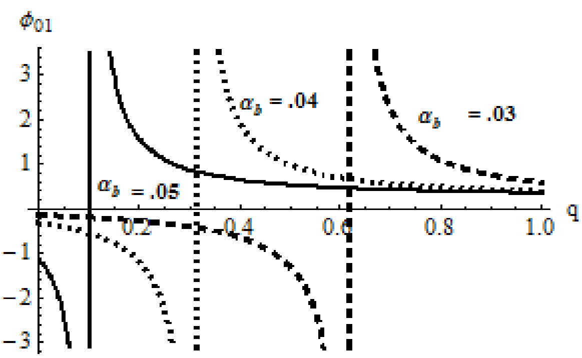

Figure 2(a). For the mode λ_1, (0<q<1) plot of amplitude ϕ_{01} as a function of q, for different three values of beam density α_b with η= 0.476, u = -0.01, v_o= 2.5, α= 0.1, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

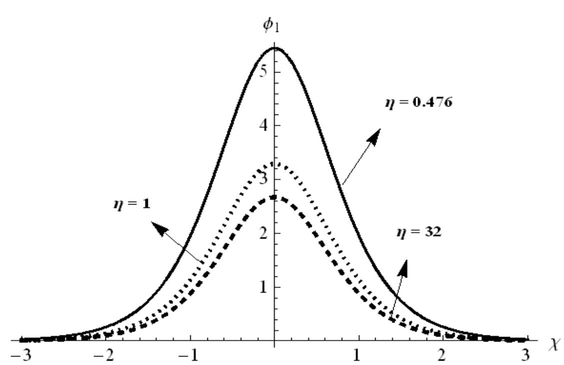

Figure 2(b). For the mode λ_1, (0<q<1), plot of soliton solution ϕ_1 as a function of χ, for three different value of η with

α=0.1, v_o= 2.5, u = -0.01, η=0.476, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

B. For the Range q>1

For given value of beam density α_b=0.1, compressive solitons are reported for modes λ_1 and λ_2, for which amplitude increases with *q*. In figure 3(a), *3D* profile of soliton solution ϕ_1 versus χ and *q* shows this behaviour. However, rarefactive solitons are reported for modes λ_3 and λ_4 respectively for which amplitude decreases with *q*.

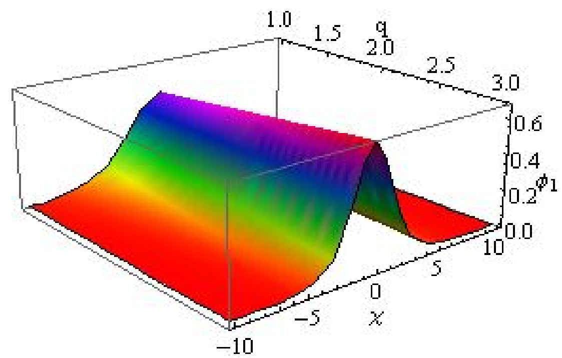

Figure 3(a). For the modeλ_1, (q>1), plot of soliton solution ϕ_1 as a function of χ and nonextensive parameter q with α=0.1, v_o= 2.5, , u = -0.01, η=0.476, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

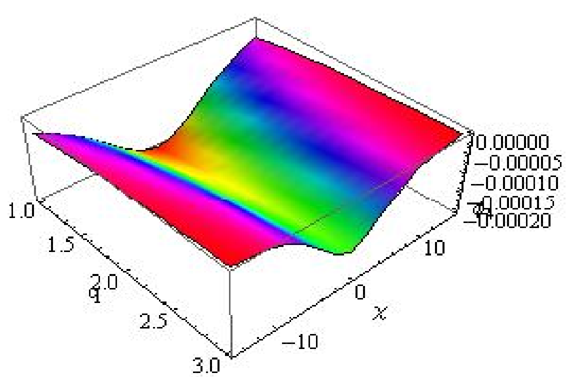

Figure 3(b). For the mode λ_4 , (q>1), plot of soliton solution ϕ_4 as a function of χ and nonextensive parameter q with α=0.1, v_o= 2.5, u=0.01, η=0.476, η_e=1/1836 and $\varepsilon_z=Z_1=Z_2=1$

This behaviour becomes clear from *3D* profile of soliton solution ϕ_4 (for mode λ_4) versus χ and *q* (see figure 3(b)). It may be mentioned that in the limit $q \rightarrow 1$ our results becomes similar to earlier ones with nonthermal parameter $\beta=0$ [37].

IV. CONCLUSION

Authors have presented a detailed numerical analysis of small amplitude ion-acoustic waves in a multicomponent plasma system containing cold positive, negative ions and electron beam. Depending upon the phase velocity, four ion-acoustic modes are obtained. The soliton solution having *sech*^{2} profile have been discussed for all the four modes in detail. The nonextensive parameter *q* and beam density α_b play a key role in characterizing the dynamics of nonlinear waves. It may be further mentioned that effect of collision and magnetic field have been neglected here. This model may be used to study the nonlinear structures in space plasma.

REFERENCES

[1] R. K. Dodd, J. C. Eilbeck, J. D. Gibbon and H. C. Morris. Solitons and Nonlinear wave equations. Academic Press, New York. 1984.

[2] C. Rebbi and G. Soliani, Solitons and particles. World Scientific Singapore, 1984, pp 331.

[3] N. J. Zabusky and M. D. Kruskal, Interaction of “solitons” in a collisonless plasma and the recurrence of initial states. Phys. Rev. Lett., 1965, pp. 240-243.

[4] H. Ikezi, R. J. Taylor and D.R. Baker, Formation and interaction of ion-acoustic solitons. Phys. Rev. Lett., 1970, 25, pp. 11-14.

[5] H. Washimi and T. Taniuti, Propagation of ion-acoustic solitary waves of small amplitude. Phys. Rev. Lett., 1966, 17, pp. 996-998.

[6] G. C. Das and S. G. Tagare, Propagation of ion acoustic waves in a multicomponent plasma. Plasma Phys., 1975, 17, pp. 1025-1032.

[7] S. G. Tagare, Effect of ion temperature on propagation of ion-acoustic solitons in a two ion warm plasma with adiabatic positive and negative ions and isothermal electrons. J. Plasma Phys., 1986, 86, pp. 301-312.

[8] S. Watanabe, Ion acoustic soliton in plasma with negative ion. J. Phys. Soc. Jpn, 1984, 53, pp. 950-956.

[9] S. G. Tagare and R.V. Reddy, Effect of ionic temperature on ion-acoustic solitons in a two-ion warm plasma consisting of negative ions and nonisothermal electrons. Plasma Phys. and Control Fusion, 1987, 29, pp. 671-676.

[10] F. B. Rizzato, R. S. Schneider and D. Dillenburg, Temperature effects on ion-acoustic solitons in plasmas

with near critical density of negative ions. Plasma Phys. and Control Fusion, 1987, 29, pp. 1127-1136.

[11] F. Verheest, Ion-acoustic solitons in a multicomponent plasmas including negative ions at critical densities. J. Plasma Phys., 1988, 39, pp. 71-79.

[12] B. C. Kalita and M. K. Kalita, Modified Korteweg-de Vries solitons in a warm plasma with negative ions. Phys. Fluids B, 1990, 2, pp. 674-678.

[13] B. C. Kalita and N. Devi, Solitary waves in a warm plasma with negative ions and drifting effect of electrons. Phys. Fluids B., 1993, 5, pp. 440-445.

[14] M. K. Mishra, R. S. Chhabra and S. R. Sharma, Obliquely propagating ion-acoustic solitons in a multicompent magnetized plasma with negative ions. J. Plasma Phys.,1994, 52, pp. 409-429.

[15] M. K. Mishra and R. S. Chhabra, Ion-acoustic compressive and rarefactive solitons in a warm multicomponent plasma with negative ions. Phys. Plasmas, 1996, 3, pp. 4446-4454.

[16] T. S. Gill, P. Bala, H. Kaur, N. S. Saini, S. Bansal and J. Kaur, Ion acoustic solitons and double layers in a plasma consisting of positive and negative ions with non-thermal electrons. The Eur. Phys. J. D., 2004, 31, pp. 91-100.

[17] Y. Nakamura and I. Tsukabayashi, Observation of modified Korteweg-de-Vries solitons in a multicomponent plasma with negative ions. Phys. Rev. Lett., 1984, 52, pp. 2356-2359.

[18] Y. Nakamura, Nonlinear and Environmental Electromagnetics, edited by H. Kikuchi (Elsevier Science Publishers B. V., Amsterdam), 1985, pp. 139-164.

[19] G. O. Ludwig, J. L. Ferreira and Y. Nakamura, Observation of ion-acoustic rarefactive solitons in a multicomponent plasma with negative ions. Phys. Rev. Lett., 1984, 52, pp. 275-278.

[20] Y. Nakamura, T. Odagiri and I. Tsukabayashi, Ion-acoustic waves in a multicomponent plasma with negative ions. Plasma Phys. and Control Fusion, 1997, 39, pp. 105-115.

[21] H. Bailung and Y. Nakumara, Oblique collision of plane ion acoustic solitons in a multicomponent plasma with negative ions. J. Plasma Phys., 1999, 61, pp. 151-159.

[22] C. Uberoi and G. C. Das, Crossover frequencies in a multicomponent plasma. Plasma Phys.,1974, 16, pp. 669-676.

[23] H. Masey, Negative ions. Cambridge University Press, Cambridge England, 1976, pp 663.

[24] W. Swider edited by J. N. Korenkev, Ionospheric Modeling. Birkhause, Basel, 1988, pp. 403.

[25] Y. Nakamura, Observation of large-amplitude ion acoustic solitary waves in a plasma. J. Plasma Phys., 1987, 38, pp. 461-471.

[26] G. C. Das, Ion-acoustic solitons and shock waves in multicomponent plasmas. Phys. Plasmas,1979, 21, pp. 2157-2165.

[27] Y. N. Nejoh and H. Sanuki, The existence of stationary ion-acoustic double layers in a plasma with electron beam. Phys. Plasma, 1997, 57, pp. 346-352.

[28] Y. N. Nejoh, Effects of electron-beam density on large-amplitude ion-acoustic waves in a plasma with trapped electrons. J. Plasma Phys., 1997 57, pp. 841-850.

[29] A. S. Shan and H. Saleem, Small amplitude ion-acoustic double layers with cold electron beam and q-nonextensive electrons. Phys. Lett., 2014, 378, pp. 795-799.

[30] Y. N. Nejoh, Large amplitude ion-acoustic waves in a plasma with a relativistic electron beam. Plasma Phys., 1996, 56, pp. 67-76.

[31] W. M. Moslem, Propagation of ion acoustic waves in a warm multicomponent plasma with an electron beam. J. Plasma Phys., 1998, 61, pp. 177-189.

[32] M. Bethomier, R. Pottelette, M. Malingre and Y. Khotyaintsev, Electron-acoustic soliton in electron beam plasma system. Phys. Plasmas, 2000. 7, pp. 2987-2994.

[33] W. F. El-Taibany and W. M. Moslem, Higher order nonlinearity of electron-acoustic solitary waves with vortex like electron distribution and electron beam. Phys. Plasmas, 2005, 12, pp. 032307.

[34] A. R. Esfandyari, I. Kourakis, and P. K. Shukla, Ion acoustic waves in a plasma consisting of adiabatic warm ions, nonisothermal electrons and a weakly relativistic electron beam: Linear and higher order nonlinear effects. Phys. Plasmas, 2008, 15, pp. 022303.

[35] S.V. Singh, G. S. Lakhiana, R. Bharuthramand, and S. R. Pillay, Electrostatic solitary structures in presence of non-thermal electrons and a warm electron beam on the auroral field lines. Phys. Plasmas, 2011, 18, pp. 122306.

[36] L. L. Yadav, R. S. Tiwari and S. R. Sharma, Ion-acoustic compressive and rarefactive solitons in a electron beam plasma system. Phys Plasmas, 1994, 1, pp. 559-566.

[37] P. Bala, T. S. Gill and H. Kaur, Localized nonlinear electrostatic structures in a multispecies space plasma. J. Phys. Conf. Ser., 2010, 208, pp.1-19.

[38] S. K. El-Labany, W. F. El-Taibany and O. M. El-Abbasy, Nonlinear electron-acoustic waves with vortex-like electron distribution and electron beam in a strongly magnetized plasma. Chaos, Solitons and Fractals, 2006 33, pp. 813-822.

[39] S. Devanandhan, S. V. Singh, G. S. Lakhina, and R. Bharuthram, Electron acoustic solitons in the presence of an electron beam and superthermal electrons. Nonlin. Processes Geophy., 2011, 18, pp. 627–634.

[40] C. Tsallis, Possible generalization of Boltzmann-Gibbs Statistics. J. Stat. Phys., 1988, 52, pp. 479- 487.

[41] L. Zhipeng, L. Liyan and D. Jiulin, A nonextensive approach for the instability of current-drive ion-acoustic waves in space plasma. Phys Plasmas, 2009, 16, pp. 072111.

[42] J. A. S. Lima, Jr. R. Silva and J. Santos, Plasma oscillations and nonextensive statistics. Phys. Rev. E, 2000, 61, pp. 3260-3267.

[43] R. Rossignoli and N. Canosa, Non additive entropies and quantum statistics. Phys. Lett. A, 1999, 281, pp. 148-153.

[44] S. Abe, S. Martinrez, F. Pennini and A. Plastino, Nonextensive thermodynamic relations. Phys. Lett. A., 2001, 281, pp. 126-130.

[45] T. Wada, On the thermodynamic stability of Tsallis entropy. Phys. Lett. A., 2002, 297, pp. 334-337.

[46] A. M. Reynolds and M. Veneziani, Rotational dynamics of turbulence and Tsallis statistics. Phys. Lett. A., 2004, 327, pp. 9-14.

[47] F. Sattin, Non-extensive entropy from incomplete knowledge of shannon entropy. Phys. Scripta, 2005, 71, pp. 443-446.

[48] J. Wu and H. Che, Fluctuation in nonextensive reaction–diffusion systems. Phys. Scripta, 2007, 75, pp. 722-725.

[49] M. Tribeche, L. Djebarni and R. Amour, Ion acoustic solitary waves in a plasma with a q-nonextensive electron velocity distribution. Phys. Plasmas, 2010, 17, pp. 4211.

[50] A. S. Bains, M. Tribeche and T. S. Gill, Modulational instability of ion acoustic waves in a plasma with q-nonextensive electron velocity distribution. Phys. Plasmas, 2011, 18, pp. 022108.

[51] N. Akhtar, W. F. Al-Taibany and S. Mahmmod, Electrostatic double layers in a warm negative ion plasma with nonextensive electrons. Phys. Lett. A, 2013, 377, pp . 1282-1289.

[52] D. K. Ghosh, G. Mandal, P. Chatterjee and U.N. Ghosh, Non planar ion acoustic solitary waves in electro positron ion plasma with warm ions, and electron and positron following q-non extensive velocity distribution. Plasma Sci., IEEE , 2013, 41, pp. 1600-1606.

[53] L. Zaghbeer, Effect of nonextensive electron and ion on dust acoustic rogue waves in dusty plasma of opposite polarity. Astro. Phys. and Space Sci., 2014, 353, pp. 2.

[54] T. S. Gill, P. Bala and A. S. Bains, Electrostatic wave structures and their stability analysis in non extensive magnetized electron-positron ion plasma, 2015, 357, pp. 63.

[55] G. Mandal and N. Y. Tanisha, Analysis of nonlinear dust-acoustic shock waves in an unmagnetized dusty plasma with q-nonextensive electrons where dust is arbitrarily charged fluid. J. Appl. Maths. and Phys., 2015, 3, pp. 103-110.

[56] S. Juneja and P. Bala, Drifting effect of electron in multi-ion plasmas with nonextensive distribution of electrons. Proceedings Int. J. Comput. Appl., IEEE (Accepted), 2015.