MHD Flow of Non- Newtonian Fluids with Volume Fraction Through Porous Medium Bounded by an Oscillating Plate

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Abstract- *In this paper we have studied MHD flow of non-Newtonian fluids through porous medium. The magnetic field used is uniform and transverse in nature. Depending upon the physical nature of the problem equations has been made. The governing*

Equations are solved by a regular perturbation method for small elastic parameter, and the expressions for velocity, temperature, concentration are obtained. The effect of the important flow parameters on the dynamics are discussed with the help of graphs. It has been observed that on increasing the non-Newtonian factor velocity of the fluid increases.

Keywords- electrically conducting, non-Newtonian fluid, oscillating plate, porous medium, transverse magnetic field

I. INTRODUCTION

Free convective flow in the presence of heat source has been always a subject of interest for scientists. The reason behind this is its application to geophysical sciences, astrophysical sciences etc.This type of flow arises due to motion of the boundary or the boundary temperature. The study of fluctuating flow is very important in the paper industry and many other technological fields. Thus many scientists have worked on such flows. Singh (2003) have analyzed the heat and mass transfer in MHD flow of viscous fluids past a vertical plate under oscillatory suction velocity. Sharma and Singh(2008)have studied the unsteady MHD-free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Singh and Gupta (2005) have studied the convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime with mass transfer.Khandelwal and Jain (2005) have discussed the unsteady MHD flow of a stratified fluid through porous medium over a moving plate in slip flow regime.Das (2011) have analyzed MHD unsteady flow of a viscous stratified fluid through a porous medium past a porous flat moving plate in the slip flow regime with heat source.

Chaudhary and Jain(2006) have found the effect of Hall current and radiation on MHD mixed convection flow of a viscoelastic fluid past and infinite plate. Sahoo(2011)have studied the unsteady two dimensional MHD flow and heat transfer of an elastic –viscous liquid past an infinite hot vertical porous surface bounded by porous medium with source and sink.

The present study is focused on the viscoelastic fluid past a vertical plate in the slip flow regime packed with uniform porous matrix in the presence of a transverse magnetic field.

The equations have been derived to find out the result and then the graphs have been made to prove and justify the result so obtained.

II. FORMULATION OF THE PROBLEM

Consider a flow of unsteady, electrically conducting and incompressible viscoelastic fluid with simultaneous heat and mass transfer near an oscillating infinite porous plate in slip flow regime with heat source and chemical reaction under the influence of a transverse magnetic field of uniform strength. The y-axis is taken along the plate in vertical direction, and x- axis is taken perpendicular to it .A uniform magnetic field of strength B_0 is applied in the direction of y-axis .Consider u, v are the velocity components. Since the plate we have taken is of infite length, all the variables are the functions of y and t.At first the plate and fluid are at rest, and then the plate is set to an oscillatory motion. The Reynolds number is assumed to be very small, and the induced magnetic field due to the flow is neglected with respect to the applied magnetic field .The pressure P in the fluid is assumed to be constant .If v^1 represents the suction velocity at the plate, the equation of continuity.

$$\frac{\partial v*}{\partial y} = 0 \tag{1}$$

Under the condition $y = 0, v *= -v_0$ everywhere.

Now the governing boundary layer equations of the flow are given by

$$(1-\varphi)\frac{\partial u*}{\partial t*} - v_0\frac{\partial u*}{\partial y*} = v\frac{\partial^2 u*}{\partial y*^2} - \sigma\frac{B_0^2 u*}{\rho(1+c^2)} - \frac{vu*}{K} + g\beta(T-T_\infty) + g\beta(C-C_\infty) - \frac{K_0}{\rho}\left(\frac{\partial^3 u*}{\partial t \partial y^{*2}} + v_0\frac{\partial^3 u*}{\partial y^{*3}}\right) \quad (2)$$

$$\frac{\partial T}{\partial t*} - v_0\frac{\partial T}{\partial y*} = \alpha\frac{\partial^2 T}{\partial y^{*2}} - S(T-T_\infty) \quad (3)$$

$$\frac{\partial C}{\partial t*} - v_0\frac{\partial C}{\partial t*} = D\frac{\partial^2 C}{\partial y^{*2}} - K_1(C-C_\infty) \quad (4)$$

Where

t: Time,

μ: Dynamic viscosity,

v :Kinematic viscosity,

α: Thermal conductivity,

p: Pressure,

g : Acceleration due to gravity,

β: Coefficient of volume expansion,

U_0: Reference velocity,

T: Dimensional temperature,

θ: Nondimensional temperature,

C: Dimensional concentration,

φ: Nondimensional concentration,

Pr: The Prandtl number,

Gr: The thermal Grashof number,

Gm: The mass Grashof number,

Sc: The Schmidt number,

M: Magnetic parameter,

S: Heat source/sink parameter,

K_l : Dimensional chemical reaction parameter,

Kc: Nondimensional chemical reaction parameter,

T_w: Temperature at the wall,

T_∞: Temperature far away from the wall,

C_w: Concentration at the wall,

C_∞: Concentration far away from the wall,

σ: Electric conductivity,

B_0: Uniform magnetic field,

v_0: Suction/injection velocity,

D: Mass diffusion ρ: Density,

K: Dimensional porosity parameter,

Kp: Nondimensional porosity parameter,

K_0: Dimensional elastic parameter,

Rc: Nondimensional elastic parameter,

n: Frequency of oscillations.

The first order velocity slip boundary condition of the problem when the plate executes linear harmonic oscillations in its own plane are given by

$$y^* = 0 : u* = U_0 e^{int} + L_1\frac{\partial u*}{\partial y*},\quad T = T_\infty,\quad C = C_\infty'$$

$$y \to \infty:\quad u \to 0,\quad T = T_\infty,\quad C = C_\infty \quad (5)$$

Where $L_1 = (2-m_1)\left(\frac{L}{m_1}\right)$, $L = \mu(\pi/2p\rho)^{1/2}$ is the mean free path, and m_1 is Maxwell's reflection coefficient.

On introducing the following non dimensional quantities

$$y' = \frac{U_0}{v}y,\ u' = \frac{u}{U_0},\ \ t' = \frac{U_0^2 t}{v}\ \ \theta' = \frac{T-T_\infty}{T_w-T_\infty},$$
$$\varphi' = \frac{C-C_\infty}{C_w-C_\infty},\ v'_0 = \frac{v_0}{U_0},\qquad n' = \frac{v}{U_0^2}n,$$
$$K'_p = \frac{KU_0^2}{v},\ R' = \frac{L_1 U_0}{v},\ S' = \frac{vS}{U_0^2},\ K'_c = \frac{vK_1}{U_0^2},$$
$$M' = \frac{\sigma B_0^2 v}{\rho U_0^2},\ G'_r = \frac{g\beta v(T_w-T_\infty)}{U_0^3},$$
$$G'_m = \frac{g\beta v(C_w-C_\infty)}{U_0^3},\ R'_c = \frac{U_0^2 K_0}{\rho v^2}\quad P_r' = \frac{v}{\alpha}$$
$$S_c' = \frac{v}{D}, \quad (6)$$

After dropping the (ʻ)

$$(1-\phi)\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - R_c\left(\frac{\partial^3 u}{\partial t\partial y^2} + v_0\frac{\partial^3 u}{\partial y^3}\right) - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p}\right)u + G_r\theta + G_m\varphi, \quad (7)$$

$$P_r\left(\frac{\partial\theta}{\partial t} - v_0\frac{\partial\theta}{\partial y}\right) = \frac{\partial^2\theta}{\partial y^2} - PrS\theta \quad (8)$$

$$Sc\left(\frac{\partial\varphi}{\partial t} - v_0\frac{\partial\varphi}{\partial y}\right) = \frac{\partial^2\varphi}{\partial y^2} - KcSc\varphi \quad (9)$$

With boundary condition

$$y = 0:\ u = e^{int} + R\frac{\partial u}{\partial y},\qquad \theta = 1,\quad \varphi = 1, \quad (10)$$
$$y \to \infty: u \to 0,\quad \theta = 0,\quad \varphi \to 0$$

Consider the following:

$$u = u_0 + R_c u_1 + O(R_c)^2,$$
$$\theta = \theta_0 + R_c\theta_1 + O(R_c)^2,$$
$$\varphi = \varphi_0 + R_c\varphi_1 + O(R_c)^2. \quad (11)$$

Substituting (11) in (7), (8) and (9) and equating like powers of R_c , we get the following.

Zeroth order equation:

$$(1-\phi)\frac{\partial u_0}{\partial t} - v_0\frac{\partial u_0}{\partial y} = v\frac{\partial^2 u_0}{\partial y^2} - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p}\right)u_0 + G_r\theta_0 + G_m\varphi_0,$$

$$P_r\left(\frac{\partial\theta_0}{\partial t} - v_0\frac{\partial\theta_0}{\partial y}\right) = \frac{\partial^2\theta_0}{\partial y^2} - PrS\theta_0$$

$$Sc\left(\frac{\partial\varphi_0}{\partial t} - v_0\frac{\partial\varphi_0}{\partial y}\right) = \frac{\partial^2\varphi_0}{\partial y^2} - KcSc\varphi_0 \quad (12)$$

First order equations:

$$(1-\phi)\frac{\partial u_1}{\partial t} - v_0\frac{\partial u_1}{\partial y} = v\frac{\partial^2 u_1}{\partial y^2} - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p}\right)u_1 + G_r\theta_1 + G_m\varphi_1 - v_0\frac{\partial^3 u_1}{\partial y^3} - \frac{\partial^3 u_0}{\partial t\partial y^2},$$

$$P_r\left(\frac{\partial\theta_1}{\partial t} - v_0\frac{\partial\theta_1}{\partial y}\right) = \frac{\partial^2\theta_1}{\partial y^2} - P, \quad (13)$$

$$Sc\left(\frac{\partial\varphi_1}{\partial t} - v_0\frac{\partial\varphi_1}{\partial y}\right) = \frac{\partial^2\varphi_1}{\partial y^2} - KcSc\varphi_1$$

The Boundary conditions are

$$y = 0:\ u_0 = e^{int} + R\frac{\partial u_0}{\partial y},$$
$$u_1 = 0,\quad \theta_0 = 1,$$
$$\theta_1 = 1 \quad \varphi_0 = 1,\quad \varphi_1 = 1, \quad (14)$$
$$y \to \infty:\ u_0 \to 0,\quad u_1 \to 0,\quad \theta_0 \to 0,$$
$$\theta_1 \to 0,\quad \varphi_0 = 0,\quad \varphi_1 = 0.$$

In order to reduce the system of partial differential equations (12) and (13) to a system of ordinary differential equations, we further introduce

$$u_0(y,t) = u_{00}(y) + u_{01}(y)e^{int},$$
$$u_1(y,t) = u_{10}(y) + u_{11}(y)e^{int},$$
$$\theta_0(y,t) = \theta_{00}(y) + \theta_{01}(y)e^{int},$$
$$\theta_1(y,t) = \theta_{10}(y) + \theta_{11}(y)e^{int},$$
$$\varphi_0(y,t) = \varphi_{00}(y) + \varphi_{01}(y)e^{int} \quad (15)$$
$$\varphi_1(y,t) = \varphi_{10}(y) + \varphi_{11}(y)e^{int}$$

Substituting (15) in (12) and (13) and equating the harmonic and the non harmonic terms, we get

$$(1-\phi)\left[\frac{\partial u_{00}}{\partial t} + \frac{\partial u_{01}}{\partial t}e^{int}\right] - v_0\left[\frac{\partial u_{00}}{\partial t} + \frac{\partial u_{01}}{\partial t}e^{int}\right] = \left[\frac{\partial^2 u_{00}}{\partial y^2} + \frac{\partial^2 u_{01}}{\partial y^2}\right] - \left[\frac{M^2}{1+c^2} + \frac{1}{K_p}\right][u_{00}(y) + u_{01}(y)e^{int}],$$

$$(1-\phi)in\,u_{01} - [v_0 u'_{00} + v_0 u'_{01}e^{int}] = u''_{00} + u''_{01}e^{int} - \left[\frac{M^2}{1+c^2} + \frac{1}{K_p}\right]u_{00} - \left[\frac{M^2}{1+c^2} + \frac{1}{K_p}\right]u_1 e^{int} + G_r\theta_{00} + G_r\theta_{01}e^{int} + G_m\varphi_{00} + G_m\varphi_{01}e^{int},$$

$$u''_{00} + v_0 u'_{00} - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p}\right)u_{00} = -G_r\theta_{00} - G_r\varphi_{00} \quad (16)$$

Similarly

$$u''_{01} + v_0 u'_{01} - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p} + (1-\phi)in\right)u_{01} = -G_r\theta_{01} - G_r\varphi_{01} \quad (17)$$

$$u''_{10} + v_0 u'_{10} - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p}\right)u_{10} = -G_r\theta_{10} - G_r\varphi_{10} - v_0 u'''_{00} \quad (18)$$

$$u''_{11} + v_0 u'_{11} - \left(\frac{M^2}{(1+c^2)} + \frac{1}{K_p}(1-\phi)in\right)u_{11} = -G_r\theta_{11} - G_r\varphi_{11} - v_0 u'''_{11} + inu''_{01} \quad (19)$$

$$Pr\frac{\partial}{\partial t}\left(\theta_{00} + \theta_{01}e^{in}\right) - v_0\frac{\partial}{\partial y}\left(\theta_{00} + \theta_{01}e^{in}\right) = \frac{\partial^2}{\partial y^2}\left(\theta_{00} + \theta_{01}e^{in}\right) - \mathrm{Pr}\left(\theta_{00} + \theta_{01}e^{in}\right)$$
$$-Pr\theta_{00}v_0 = \theta''_{00} - PrS\theta_{00}$$
$$\theta''_{00} + Pr\theta_{00}v_0 - PrS\theta_{00} = 0 \quad (20)$$

And

$$\mathrm{Pr}\,in - v_0 Pr\,\theta'_{01} = \theta''_{01}e^{in} - PrS\theta_{01}$$

Or

$$\theta''_{01} + v_0 Pr\theta'_{01} - (inPr + PrS)\theta_{01} = 0 \quad (21)$$

Similarly making substitution in all equations, we get

$$\theta''_{10} + Pr\theta_{10}v_0 - PrS\theta_{10} = 0 \quad (22)$$

$$\theta''_{11} + v_0 Pr\theta'_{11} - (inPr + PrS)\theta_{11} = 0 \quad (23)$$

$$\varphi''_{00} + v_0 Sc\varphi_{00}v_0 - KcSc\varphi_{00} = 0 \quad (24)$$

$$\varphi''_{01} + v_0 Sc\varphi'_{01} - (inSc + KcSc)\varphi_{01} = 0 \quad (25)$$

$$\varphi''_{10} + v_0 Sc\varphi_{10} - KcSc\varphi_{10} = 0 \quad (26)$$

$$\varphi''_{11} + v_0 Sc\varphi'_{11} - (inSc + KcSc)\varphi_{11} = 0 \quad (27)$$

With boundary conditions

$y = 0: u_{00} = R\frac{\partial u_{00}}{\partial y}, u_{01} = 1 + R\frac{\partial u_{01}}{\partial y},$
$u_{10} = 0,\ u_{11} = 0,\ \ \theta_{00} = 1,$
$\theta_{01} = 0,\ \theta_{10} = 1\ \ \theta_{11} = 1,$
$\varphi_{00} = 1,\ \ \varphi_{01} = 0,\ \ \varphi_{10} = 0,\ \ \varphi_{11} = 0,$
$y \to \infty:\ u_{00} \to 0,\ u_{01} \to 0,\ \ u_{10} \to 0,$
$u_{11} \to 0,\ \theta_{00} \to 0,\ \ \theta_{01} \to 1,$
$\theta_{10} \to 0,\ \theta_{11} \to 0,\ \ \varphi_{00} \to 1,$
$\varphi_{01} \to 0,\ \varphi_{10} \to 0,\ \ \varphi_{11} \to 1,$

The solutions of (16) to (27) using the boundary conditions we get
$u_{00} = A_1e^{-a_1y} + A_2e^{-a_2y} + A_3e^{-a_3y},$
$u_{01} = A_4e^{-a_4y},$
$u_{10} = A_5e^{-a_1y} + A_6e^{-a_2y} + A_7e^{-a_3y},$
$u_{11} = A_8e^{-a_4y},$
$\theta_{00} = e^{-a_1y},$
$\theta_{01} = \theta_{10} = \theta_{11} = 0,$
$\varphi_{00} = e^{-a_2y},$
$\varphi_{01} = \varphi_{10} = \varphi_{11} = 0,$

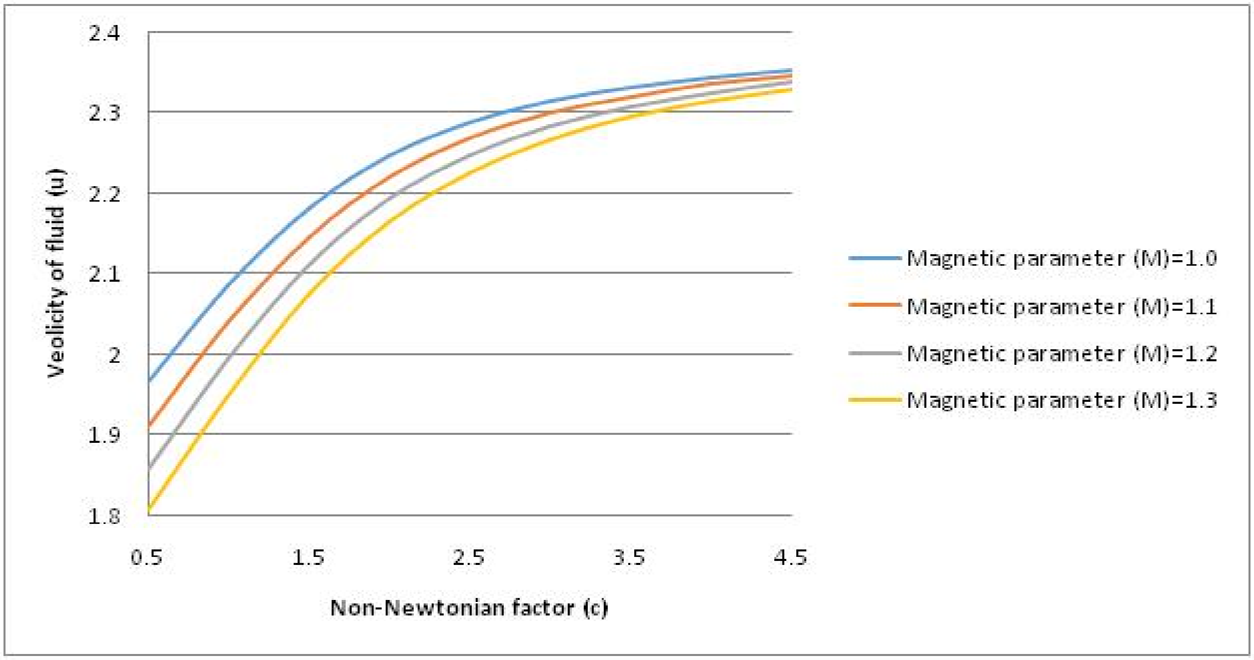
Hence the velocity , temperature and concentration are given by
$u = u_0(y,t) + Rc\, u_1(y,t),$
And
$u = u_{00}(y,t) + Rc\, u_{01}(y)e^{int} + Rc(u_{10}(y) + Rc\, u_{11}(y)e^{int}),$

$$u = A_1e^{-a_1y} + A_2e^{-a_2y} + A_3e^{-a_3y} + A_4e^{-a_4y}(\cos nt + i\sin nt) + Rc[A_5e^{-a_1y} + A_6e^{-a_2y} + A_7e^{-a_3y} + A_8e^{-a_4y}(\cos nt + i\sin nt)] \quad (28)$$

And
$\theta = \theta_0 + Rc\theta_1,$
$\theta = \theta_{00} + \theta_{01}e^{int} + Rc(\theta_{10} + \theta_{11}e^{int}),$
$$\theta = e^{-a_1y} \quad (29)$$
$\varphi = \varphi_0 + Rc\varphi_1,$
$\varphi = \varphi_{00} + \varphi_{01}e^{int} + Rc(\varphi_{10} + \varphi_{11}e^{int}),$
$$\varphi = e^{-a_2y} \quad (30)$$

Where
$a_1 = \frac{1}{2}\left[v_0Pr + \sqrt{v_0^2Pr^2 + 4SPr}\ \right],$
$a_2 = \frac{1}{2}\left[v_0Sc + \sqrt{v_0^2Sc^2 + 4KcSc}\ \right],$
$a_3 = \frac{1}{2}\left[{v_0}^2 + \sqrt{v_0^2 + 4Q}\ \right],$
$a_4 = \frac{1}{2}\left[{v_0}^z + \sqrt{v_0^2 + 4\{Q + in(1-\phi)\}}\ \right],$
$A_1 = -\frac{Gr}{a_1^2 - a_1v_0 - Q},$
$A_2 = -\frac{Gr}{a_2^2 - a_2v_0 - Q},$
$A_3 = -\frac{1}{1+a_3R}[(a_1R+1)A_1 + (a_2R+1)A_2],$
$A_4 = -\frac{1}{1+a_4R},$
$A_5 = -\frac{a_2^3A_2v_0}{a_1^2 - a_1v_0 - Q},$
$A_6 = -\frac{a_2^3A_2v_0}{a_2^2 - a_2v_0 - Q},$
$A_7 = A_5 + A_6 - \frac{a_3^3A_3v_0y}{v_0 - 2a_3},$
$A_8 = \frac{(-v_0a_4^3A_4 - ina_4^2A_4)y}{v_0 - 2a_4}.$

Figure 1
Velocity of fluid (u) and Non-Newtonian factor (c)



III. RESULT AND DISCUSSION

Figure 1 show the change in the velocity of fluid with changes in the volume fraction .It has been observed that on increasing the volume fraction of the dust particle the velocity of the fluid increases. The graph initially increases in exponentially and then becomes parallel to the axis of non-Newtonian factor .The result observed by the graph is obvious because some of the terms involving NonNewtonian factor comes in the exponential form and some are in simple form .Hence the combined effect of all terms can be seen by the above graph.

It is further observed that on increasing the magnetic field of the system the initial velocity of the fluid decreases. This decrease in velocity is due to Lenz Law. It has also been found that the magnetic field parameter has a very important effect on the velocity. On close observations it can be seen that the lines for different values of magnetic field are equally spaced initially but they become closer at the end of the graph. Thus as fluid becomes nonNewtonian the effect of magnetic field and other factors decreases.

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