Adaptation of Weibull Distribution on Channel Modeling for Satellite Communication Links above 10GHZ

D.Rajesh[1], R.Kalyan[2]
[1, 2]Department of ECE
[1, 2] SITS, Tirupati, Andhra Pradesh, India

Abstract- *Satellite Communication Networks in modern days has higher frequencies above 10GHZ.Frequencies more than 10GHZ causes rain attenuation. In this paper, a channel model is defined to synthesize rain attenuation of 10GHZ and above. Previously, for satellite communication networks, Maseng-Bakken(M-B)model is used which is a time series synthesizer proposed based on Stochastic Differential equations which assumes that rain attenuation is determined by lognormal distribution and rate of change in rain attenuation proportional to its instantaneous values. In this paper, a new model is proposed that modifies the existing Maseng-Bakken(M-B) model and generates rain attenuation time series based on Weibull Distribution. It is based on first order stochastic differential equations and rain attenuation is introduced on the incline path as a Weibull based stochastic process. Theoretically, the probabilities of hitting time random variables are represented in this model. Hitting time statistics are useful in the design of Fade Mitigation Techniques(FMT).The proposed synthesizer is verified in terms of probability and theoretical CCDF(Complementary Cumulative Distribution Function) of hitting times and comparison of these are derived from simulations which are discussed in Numerical results of Weibull Distribution.*

Keywords- Maseng-Bakken(M-B) model, Stochastic Differential Equations(SDEs), Adaptive Coding and Modulation (ACM) Technique, Power Control, Fade Mitigation Techniques (FMTs), Complementary Cumulative Distribution Function (CCDF), Weibull Based Stochastic Dynamic Model, Hitting Time Statistics.

I. INTRODUCTION

An operation on satellite communications needs high frequency bands such as ku, Ka and Q/V. But these bands are scarcely available and the needs for these bands are high. Frequencies that are operating above 10GHZ effects with rainfall, which is one of the fading mechanism that causes highest attenuation in various atmospheric conditions. The implementation of a fixed power margin as a measure for rain attenuation is not a optimal solution in case of higher rain attenuations. The solution over fixed power margins is solved by introducing Fade Mitigation Techniques (FMT) into the system. FMT includes other techniques like Adaptive Coding Modulation (ACM) and power control. FMT uses time series synthesizers for evaluation. In some cases where the experimental rain attenuation time series are not presented, system evaluation is achieved by using time series synthesizers. Previously, M-B model utilizes time series synthesizers that are based on stochastic differential equations (SDEs) in which rain attenuation uses a lognormal distribution which is proportional to its instantaneous values. Weibull distribution is another way to describe rain attenuation. It is one of the better prediction models when compared to other models. Hitting time statistics are used as parameters to represent rain attenuation dynamics. Hitting time statistical parameters are used define calculation of rain attenuation dynamic parameter and optimizing FMT. This paper introduces a novel rain attenuation time series synthesizer and calculates it's hitting time statistical distribution. Weibull distribution uses first order statistics of rain where the rate of change of rain attenuation is relative to instantaneous values of rain attenuation. This paper also helps in development and design of future generation FMTs and a new set of protocols for broadband satellite networks which are operating at higher frequencies like 10GHZ.

II. MASENG- BAKKEN (M-B) MODEL

In case of fixed terrestrial radio systems that are operating at higher frequencies normally greater than 10GHZ in tropical and equatorial areas effect with rain attenuation. In such cases the frequency propagation are said to be severe, signal are out of range and performance standards are difficult to predict. FMTs are used to avoid these abnormal propagation situations. Implementation of cumulative distribution in rain attenuation is commonly used in designing of links. But it is not enough to optimize and design a FMT. Normally propagation of signal depends on Fade durations and Fade slope statistics which are also needed for a FMT.A time series of propagation impairments are introduced in order to estimate Fade duration and Fade slope in the system for simulation. For terrestrial links in tropical areas, a rain attenuation time series uses Maseng-Bakken principle. M-B model uses two long term rain attenuation time series synthesizers. One is EMB, which is called as Enhanced MB model used in satellite systems at temperate climatic conditions. another is TMB used in Terrestrial links.

MASENG-BAKKEN AND ENHANCHED MASENG-BAKKEN MODELS (EMB)

Maseng-Bakken model proposed a stochastic model of rain attenuation which is based on the first order Markov theory. It is applied to generate a time series for rain attenuation in satellite communication systems that are operating with ku-band or above. In M-B model there are two hypotheses for rain attenuation, one is A_{rain} and another hypotheses is long-term distribution of rain attenuation which is log-normal determined by the mean "m" and the standard deviation "σ". The transformation of rain attenuation into a stationary Markov process using the nonlinear transformation of first order is:

$$X = (\ln A_{rain} - m)/\sigma \qquad (1)$$

β, is used as a parameter to describe the dynamics of the rain attenuation. The original model gives only the information about the periods of rain that are synthesized. Therefore, prior to the retrieval of parameters, attenuation value which are below some threshold must be eliminated from the time series while experimenting.

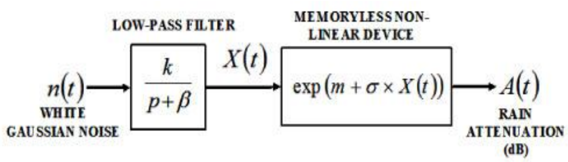

Fig. 1. Principle of the Maseng-Bakken model

EMB model is used for synthesizing both periods of rain and no rain. For a EMB, the input parameters are obtained from the attenuation time series which is not in the given range of threshold level. The CCDF based on curve fitting methods is used to determine the lognormal parameters m and σ.EMB model also uses fourth parameter, attenuation offset Aoff . In order to advance the performance of the model Aoff is subtracted from the synthesized time series.

TERRESTRIAL ENHANCHED MASENG-BAKEN MODEL

TMB model uses same parameters as EMB model except that the extraction procedures of parameters are different. CCDF in used to define the parameters m and σ and also minimizes the RMS errors in dB both in theory and experiments of a synthesized attenuation. With CCDF parameters m and σ, the attenuation value Aoff is determined corresponding to p% of time. The value of p is set to 10%, which is upper limit range of time percentage used in curve fitting. The value of p is derived from the experiments on rain attenuation CCDFs of the links.

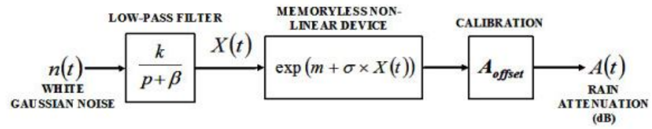

Fig. 2. Principle of the Enhanced Maseng-Bakken model

III. WEIBULL BASED STOCHASTIC DYNAMIC MODEL

Based on the experimental values, it was found that the Root Mean Square (RMS) value of relative error was 13.8% for lognormal distribution and 13.47% for weibull distribution. Therefore weibull distribution provides better estimation of extended probability than lognormal in rain attenuation. The PDF (Probability Density Function) of Weibull distribution is:

$$p_X(x) = \frac{\nu}{w} x^{\nu-1} e^{-\frac{x^{\nu}}{w}}, \; x > 0 \qquad (1)$$

Where w and ν are the parameters of Weibull distribution, and it is greater than 0.The SDE for the rain attenuation modeling and A(t) is given:

$$dA(t) = K_1(A(t)) \cdot dt + \sqrt{K_2(A(t))}\, dB(t) \qquad (2)$$

Where Brownian Motion is B(t) and the Brownian increments are dB(t).The drift coefficient is K1(A(t)) and the diffusion coefficient is K2(A(t)) of the Stochastic Differential Equation. The rate of change in rain attenuation is measured proportional to its instantaneous values of rain attenuation, which is established with its experimental data. Therefore, diffusion coefficient:

$$K_2(A(t)) = \frac{2 d_a w}{\nu} A^2(t) \qquad (3)$$

Where, the dynamic parameter of rain attenuation is d_α. The d_α values are calculated either from local rain attenuation measurements or from a set which is equivalent to the proposed values of 2 $.10^{-4}$, according to the recommendations of ITU-R. P.1853-1. Based on the considerations of static distribution of rain attenuation stochastic process, Weibull distribution determines A(t), and the drift coefficient are derivated from the solution of:

$$p_{st}(A) = \frac{C}{K_2(A)} \exp\left\{ 2 \int \frac{K_1(y)}{K_2(y)} dy \right\} \qquad (4)$$

With C as normalization constant. The static PDF of rain attenuation is given by $p_{st}(A)$. Assuming that the former is the theoretical PDF of the Weibull equation (1), where the drift coefficient is given by:

$$K_1(A(t)) = d_a A(t)\left(w + \frac{w}{\nu} - A^{\nu}(t)\right) \qquad (5)$$

Consequently, the SDE of the form of equation (2) has been defined with two coefficients given in equation(3) and equation (5).The solution for SDE is:

$$A(t) = \frac{A_0 \cdot e^{\sqrt{\frac{2 d_a w}{\nu}} \cdot B(t) + d_a \cdot w \cdot t}}{\left(1 + \nu d_a \cdot A_0^{\nu} \int_0^t e^{\nu \sqrt{\frac{2 d_a w}{\nu}} \cdot B(s) + \nu \cdot d_a \cdot w \cdot s} ds\right)^{1/\nu}} \qquad (6)$$

Where the Brownian Motion is B(t) and A_0 is the starting value of rain attenuation, which is a smaller value such as 0.5dB. Therefore, using equation (6), rain attenuation time series can be generated. By using equation (6),a MATLAB file can be determined for the rain attenuation time series synthesis.

IV. HITTING TIME STATISTICS

The time required for a stochastic process to reach minimum or maximum threshold values is said to be Hitting Time, defined as A_{min} or A_{max}, respectively. For a given value A0 of a stochastic process, A0 lies in between minimum and maximum values of A. Amin ≤ A0 ≤ Amax, with the initial time instance $t=t_0$.Because hitting time is related to a stochastic process, the prior is a random variable.

In this segment, hitting time CCDF are computed analytically for a defined time series synthesizer and given in equation (6) and described by equation (2) with coefficients derived from equation(3) and equation(5). (τh) is a moment generation function of the hitting time which gives the solution for the differential equations

$$Tu - \lambda u = 0 \text{ on } D$$
$$u = 1 \text{ at } A_0 = A_{\min}, A_0 = A_{\max} \qquad (7)$$

Where D ≡ (Amin, Amax) and Infinitesimal operator T is given by:

$$Tu(A_0) = \frac{1}{2} K_2(A_0) \frac{d^2}{dA_0^2} u(A_0) + K_1(A_0) \frac{d}{dA_0} u(A_0) \qquad (8)$$

The function u from the solution of equation (7) is:

$$u = C_1 \Lambda_1(A_0) + C_2 \Lambda_2(A_0) \qquad (9)$$

With

$$C_1 = \frac{\Lambda_2(A_{\max}) - \Lambda_2(A_{\min})}{\Lambda_2(A_{\max})\Lambda_1(A_{\min}) - \Lambda_2(A_{\min})\Lambda_1(A_{\max})}$$
$$C_2 = \frac{\Lambda_1(A_{\min}) - \Lambda_1(A_{\max})}{\Lambda_2(A_{\max})\Lambda_1(A_{\min}) - \Lambda_2(A_{\min})\Lambda_1(A_{\max})}$$
$$\Lambda_{1,2}(x) = \exp\left(\frac{2}{\nu\gamma^2} x^{\nu}\right) p_{1,2}(x)$$
$$p_{1,2}(x) = x^{\nu\kappa_{1,2}} F_{11}(a_{1,2}, b_{1,2}; -\frac{2x^{\nu}}{2w}), \kappa_{1,2} = \frac{-\frac{\nu}{2} \pm \frac{\nu}{2}\sqrt{1 + \frac{4}{\nu w}\lambda}}{\nu}$$
$$\gamma = \sqrt{2\frac{w}{\nu}}, a_{1,2} = \kappa_{1,2} + 2, b_{1,2} = 2\kappa_{1,2} + 2 \qquad (10)$$

Where the statistical parameter of Weibull distribution is represented as ν and Krummers function of the first kind is F11(.,.;.). The CCDF of hitting time (τh) is:

$$P\left[\tau_h \geq \tau | A(t_0) = A_0\right] = F(A_{\min}, A_{\max}, A_0, \infty) - F(A_{\min}, A_{\max}, A_0, \tau) \qquad (11)$$

In Which F is a Inverse Laplace Transform (ILT) of:

$$U(A_{\min}, A_{\max}, A_0, \lambda) = \frac{u(A_{\min}, A_{\max}, A_0, \lambda)}{\lambda} + \frac{F(A_{\min}, A_{\max}, A_0, 0)}{\lambda} \qquad (12)$$

The function u is chosen between minimum and maximum thresholds of attenuation and calculated by equation (9) and the ILT of equation (12) gives a function F(.) and the hitting times based on CCDF can then be calculated by equation(11).

FMTs of Satellite communication depends on dynamics of rain attenuation which are determined from hitting time statistics. These techniques are time diversity, adaptive coding and modulation, and the power control schemes. The above proposed measurements can be useful in short-term prediction of rain attenuation and optimizing the design of FMTs with energy and efficiency in spectral. For example, in satellite protocols where time diversity is considered, data which is retransmitted over a waiting period and where the initial time instance rain attenuation is said to be very high because of augmentation of de-correlation of rain attenuation with respect to time. Statistical hitting times helps a system to develop time domain as diversity means in its design.

GRAPHS FOR SATELLITE COMMUNICATION LINKS BASED ON WEIBULL DISTRIBUTION WITH FREQUENCY ABOVE 10GHZ

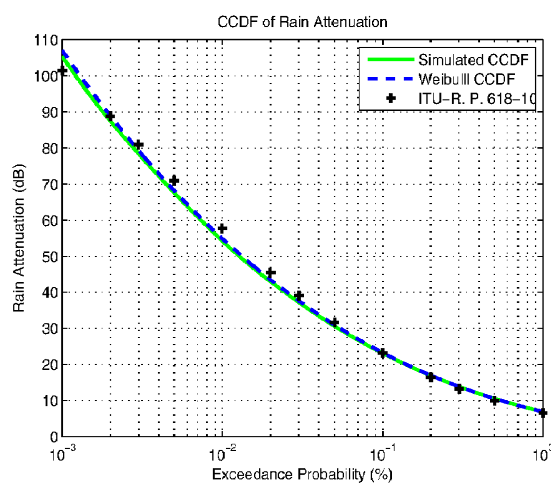

Fig. 1. CCDF of rain attenuation predicted by the ITU-R. P. 618-10 model (crosses), Weibull fitted distribution (dashed line), derived from simulation time series (solid line).

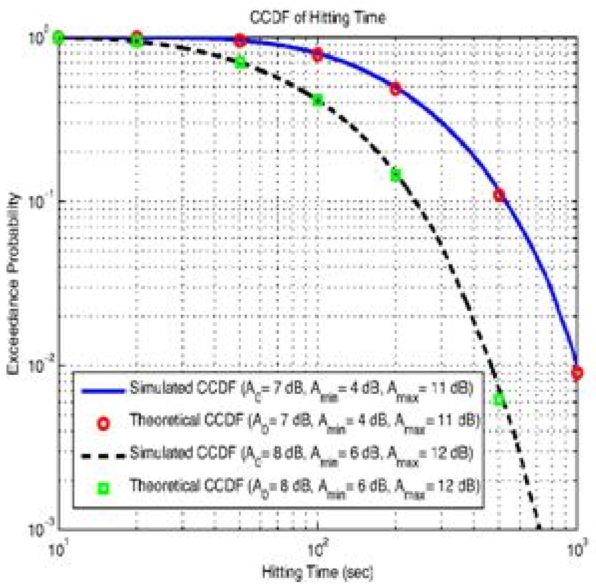

Fig. 2. CCDF of hitting time of rain attenuation for a hypothetic link in Athens, GR. for two different thresholds (A_0, A_{min}, A_{max}): (7, 4, 11) and (8, 6, 12) derived from simulations (solid line and dashed line, respectively) and the theoretical expressions (circles and squares, respectively).

V. DISCUSSION ON NUMERICAL RESULTS OF WEIBULL DISTRIBUTION

The proposed synthesizer from Weibull distribution is compared and validated with the predictions of ITU-R. P.618-10 [17], in conditions of the rain attenuation exceeding probability. The hypothetical expressions of hitting time statistics are compared with the statistical hitting times which are derivated from set of time series synthesizer from equation (6).once the fitting of CCDF for Weibull distribution with the predicted CCDF of rain attenuation is completed, the statistical parameters w and v of the Weibull distribution are calculated and compared. The parameters w and v are used for the invention of rain attenuation time series using equation (6).From the Fig.1, three curves with extended probability of rain attenuation are shown which follows Weibull distribution. The three curves which are represented are one from the Weibull CCDF, distribution, one from ITU-R. P. 618-10[17] and one from the time series synthesizer. The frequency of the operating link is 40GHz and the angle of elevation 20o for a Earth station which is situated at a mid-latitude region at Europe. The Weibull distribution statistical parameters are w=0.41497 and v=0.3347. In the Figure, it can be observed that the result of CCDF from the time series matches exactly with the CCDF Weibull. In Fig.2, shows CCDF hitting time statistics for a hypothetic link in Athens, GR. where the parameters of statistical Weibull distribution for rain attenuation is calculated by considering the predictions made by ITU-R. P. 618-10 [17]. The angle of elevation of the link is 30o and the operating frequency 25 GHz.

VI. CONCLUSION

In this paper, In case of fixed satellite communication links which are operating above 10GHz, a novel channel model called Weibull distribution has been presented. With the similar results to the lognormal distribution, Weibull distribution can model the extended probability of rain attenuation. Weibull distribution modeling is based on first order stochastic differential equations with diffusion coefficient proportional to the square of the given rain attenuation instantaneous values. The expressions for extended probability of hitting times of rain attenuation in a proposed synthesizer are calculated analytically and applications based on weibull distribution to FMTs are determined. At last it is verified that SDE reproduces the extended probability of the Weibull Distribution in which the parameters are given input to the model and the CCDF of hitting time is formulated analytically and reproduced from the simulators. Now these Synthesizers are used in satellite systems for accurate evaluation and assessment of performance of FMTs.

REFERENCES

[1] “Satellite communications at ku, ka and V bands: propagation impairments and mitigation techniques,”by A. D. Panagopoulos, P.-D. M. Arapoglou, and P. G. Cottis, IEEE Commun. Surveys Tuts., pp. 2–14, 2004.

[2] S. N. Livieratos, V. Katsambas, and J. D. Kanellopoulos, “A global method for the prediction of the slant path rain attenuation statistics,”J. Electrom. Waves and Applic., vol. 14, no. 5, pp. 713–724, 2000.

[3] “Long-term rain attenuation probability and site diversity gain prediction formulas” ,A. D. Panagopoulos, P.-D. M. Arapoglou, J. D. Kanellopoulos, and P. G. Cottis, IEEE Trans. Antennas Propag., vol. 53, no. 7,pp. 2307–2313, 2005.

[4] “Rain attenuation hitting time statistical distributions: application to fade mitigation techniques of future satellite communication systems,” by C. I. Kourogiorgas, S. A. Kanellopoulos, A. D. Panagopoulos,S. N. Livieratos, G. E. Chatzarakis, and J. D. Kanellopoulos, in 2012 ESTEL Conference.

[5] “A modified synthesis procedure for 1st order stochastic differential equations for the simulation of base band random processes signal processing,”. A. Kanellopoulos, G. Fikioris, A. D. Panagopoulos, and J. D. Kanellopoulos, J. Signal Process., vol. 87, no. 12, pp.

3063–3074, 2007.

[6] "Hitting Time Distribution of Weibull-based Rain Attenuation Synthesizer: An Application to Satellite Communications FMTs",by Charilaos I. Kourogiorgas, Mobile Radio Communication Laboratory School of Electrical and Computer Engineering; National Technical University of Athens