Study and Design of Sierpinski Carpet Fractal Antenna

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Abstract- *Fractal antennas are multiband, large gain and low profile antennas that are used for wireless applications. Fractal antennas can give results similar to a larger sized antenna in spite of having a small and compact size due to self iterative geometry. An iterative geometry refers to the technique of subdividing a shape into smaller copies of itself. A Sierpinski Fractal Antenna is a planar fractal antenna that is made by subdividing a square into same shaped smaller copies. In this paper a multiband Sierpinski Carpet Fractal Antenna has been designed and its parameters have been studied. It has been designed for a frequency range of 3 GHz to 6 GHz. The antenna is intended for wireless applications like GSM.*

Keywords- Antenna, Fractal, Sierpinski, Koch, Minkowski, multiband, wideband, wireless applications.

I. INTRODUCTION

Communication is an ever improving and progressing field and today communication systems require antennas with the following desired properties:

1. Multiband
2. Wideband
3. Low Profile
4. Small or compact in size

These demands are fulfilled by Fractal antennas. The term Fractal refers to broken or irregular fragments and this term was coined by Mandelbrot[1] to describe a family of complex shapes that had a property of self iteration, i.e. self repeating with scaling.

II. GEOMETRIES FOR FRACTAL ANTENNA

Fractal antennas are designed using the self iterative technique that is using similar self copies. Some of the designs discussed are [2-3]:

1. Sierpinski Gasket
2. Sierpinski Carpet
3. Koch Snowflake
4. Hilbert Curve
5. Minkowski Island

A. Sierpinski Gasket

The Sierpinski Gasket, also called the Sierpinski Triangle or the Sierpinski Sieve, is a fractal and attractive fixed set named after the Polish mathematician Wacław Sierpinski who described it in 1915. The construction of Sierpinski Gasket starts with an equilateral triangle with a base parallel to the horizontal axis, as shown in figure 2.1. Then further steps are followed, as shown in figure 2.1, to get the final design as seen in [2] and [4].

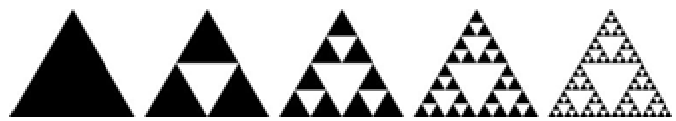

Fig 2.1 Stepwise design of Sierpinski Gasket

B. Sierpinski Carpet

The Sierpinski carpet is a plane fractal first described by Wacław Sierpinski in 1916. The construction of the Sierpinski carpet begins with a square. The square is cut into 9 congruent sub squares in a 3-by-3 grid, and the central sub square is removed as shown in figure 2.2. The same procedure is then applied recursively to the remaining 8 sub squares, and this can be repeated for multiple iterative levels as seen in [2] and [5].

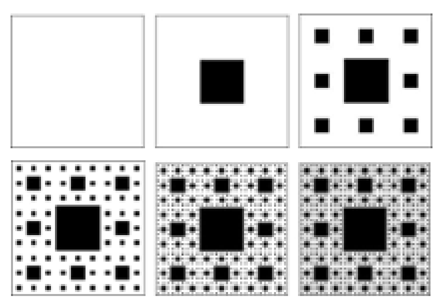

Fig 2.2 Sierpinski Carpet Antenna up to five iterations

C. Koch Snowflake

The Koch Snowflake, also known as the Koch star and Koch Island, is a mathematical curve and one of the earliest fractal curves to have been described. The process of designing the antenna is depicted in figure 2.3.

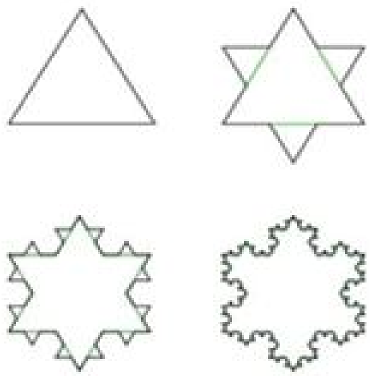

Fig 2.3 Koch snowflake Antenna up to three iterations

D. Hilbert Curve

A Hilbert curve (also known as a Hilbert space-filling curve) is a continuous fractal space-filling curve first described by the German mathematician David Hilbert in 1891[6] as a variant of the space-filling curves discovered by Giuseppe Peano in 1890[7]. Its design and algorithm for iteration can be found in [8]. Figure 2.4 shows a Hilbert Curve up to 5 iterations.

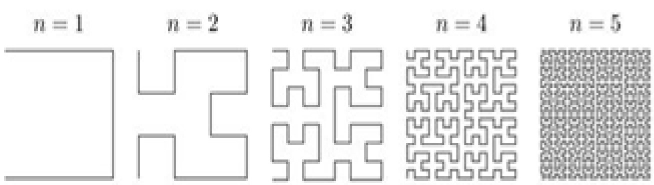

Fig 2.4 Hilbert curve antenna up to five iterations

E. Minkowski Island

Another type of a fractal antenna design is Minkowski Island. As the number of iterations of the Minkowski fractal increases, the resonant frequencies increase and the bandwidth of each single band increases. Also, it is found that increasing the number of iterations of the fractal antenna causes a decrease in the antenna gain, input impedance, and voltage standing wave ratio, and it enhances the antenna matching [9]. A Minkowski Island design has been depicted in figure 2.5.

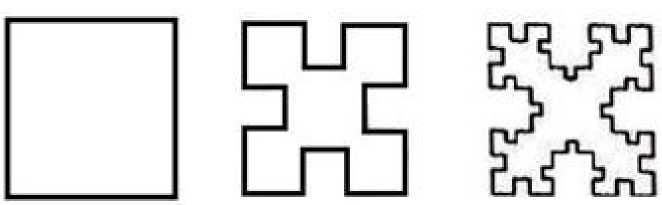

Fig 2.5 Minkowski Island Fractal Design

Depending upon the requirement and geometry any of the designs can be implemented for various applications.

III. ITERATIVE FUNCTION SYSTEM ALGORITHM

In order for an antenna to work equally well at all frequencies, it must satisfy two criteria: it must be symmetrical about a point, and it must be self-similar, having the same basic appearance at every scale: that is, it has to be fractal. The shape of the fractal is formed by an iterative mathematical process. This process can be described by an iterative function system (IFS) algorithm, which is based upon a series of affine transformations [2]. An affine transformation in the plane ω can be written as [9]:

$$\omega\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Ax + t = \begin{pmatrix} r_1\cos\theta_1 & -r_2\sin\theta_2 \\ r_1\sin\theta_1 & r_2\cos\theta_2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

here x_1 and x_2 are the coordinates of point x. If $r_1=r_2=r$ with $0<r<1$, and $\theta_1=\theta_2=\theta$, the IFS transformation is a contractive similarity (angles are preserved) where r is the scale factor and θ is the rotation angle. The column matrix t is just a translation on the plane.

Designing can also be done using a scaling factor. For example in fig 2.2, design of Sierpinski Carpet antenna, a scaling factor of 1/3 has been taken which means that is with every successive iteration, coordinates and lengths get scaled by a factor 1/3.

IV. ANTENNA DESIGN SPECIFICATION

Various parameters for the design of a Sierpinski Carpet antenna can be calculated using the formulas mentioned below [10]:

Width;

$$W = \frac{v_o}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}}$$

Effective dielectric;

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r+1}{2}\left(\frac{1}{\sqrt{1+12d/W}}\right)$$

Fringing Field;

$$\Delta l = 0.412d\,\frac{\left(\varepsilon_{eff}+0.3\right)\left(\frac{W}{d}+0.262\right)}{\left(\varepsilon_{eff}-0.258\right)\left(\frac{W}{d}+0.813\right)}$$

Length;

$$L = \frac{v_0}{2 f_r \sqrt{\varepsilon_{eff}}} - 2\Delta l$$

Where,

v = Velocity of light in free space.
fr = Operating resonant frequency.
ε = Dielectric constant of the substrate used.
εeff = effective dielectric constant
d = height of the substrate.

The generalized formulas for iteration n are as follows:

Nn = number of black box.
Ln = ratio for length.
An = ratio for the fractal area after the nth iteration.
n = iteration stage number.
Nn = 8n

$$L_n = \left\{\frac{1}{3}\right\}^n$$

$$A_n = \left\{\frac{8}{9}\right\}^n$$

Using the above mentioned formulas parameters for a Sierpinski Carpet antenna were found. Using HFSS a model was implemented and simulated with the antenna shown in figure 3.1.

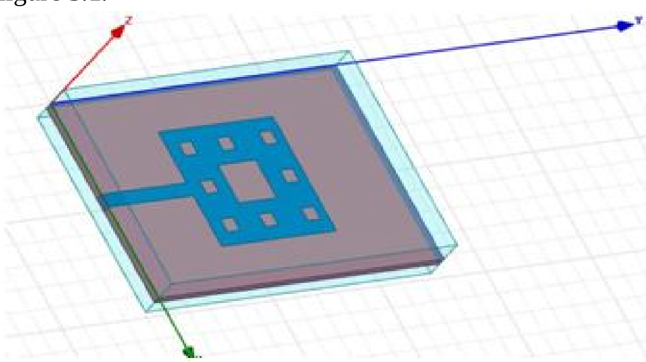

Fig 3.1 Screenshot of Antenna Designed using HFSS used to implement the proposed antenna. Results show that the proposed antenna has low return loss and value of VSWR

V. RESULTS

The designed antenna was simulated using HFSS and results were achieved. Low values of return loss and VSWR were obtained at the following frequencies:

- 3.2 GHz
- 3.6 GHz
- 4.1 GHz
- 5.2 GHz
- 5.9 GHz

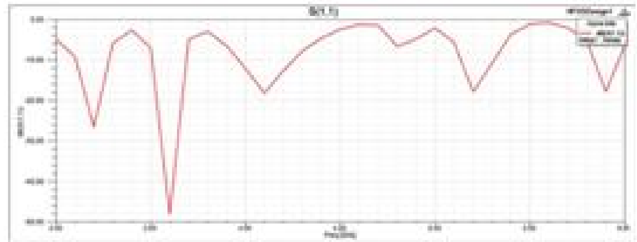

Fig. 4.1 Return Loss

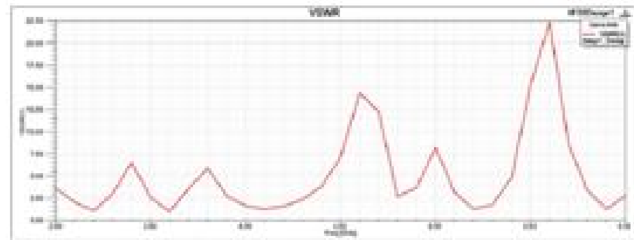

Fig 4.2 VSWR

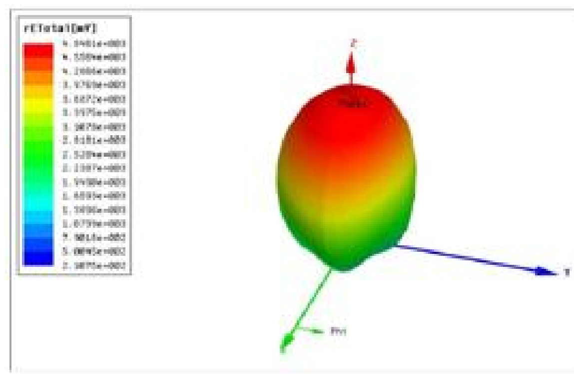

Fig 4.3 Radiation Pattern

VI. CONCLUSION

A Sierpinski Carpet Antenna has been presented in this paper along with a brief introduction to different types of Fractal Antenna designs being currently used. An antenna simulation tool called HFSS or High Frequency Structural Simulator was over multiple frequencies. Hence this can be used for practical purposes to achieve more efficient and better communication.

In today's world there is an ever growing demand of small compact and efficient wideband and multiband antennas. Fractal antennas can prove to be a solution to all these requirements.

VII. ACKNOWLEDGEMENT

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